

Small-Angle X-ray Scattering – Interpretation of the results: “What can be obtained from the scattering curve?”

András Wacha

Research Centre for Natural Sciences, Hungarian Academy of Sciences

Outline

Recapitulation

Self-assembled systems

- Ordered phases

 - Multilamellar and hexagonal phases

- Unilamellar vesicles

- Micellar systems

Continuous – hierarchical – systems

- Anisotropy and porosity of activated carbons

- In situ experiments on a gold-cysteine self-assembling nanocomplex

Particulate systems

- Size distribution of SiO₂ nanoparticles

- Proteins – biological macromolecules

Summary

Outline

Recapitulation

Self-assembled systems

- Ordered phases

 - Multilamellar and hexagonal phases

- Unilamellar vesicles

- Micellar systems

Continuous – hierarchical – systems

- Anisotropy and porosity of activated carbons

- In situ experiments on a gold-cysteine self-assembling nanocomplex

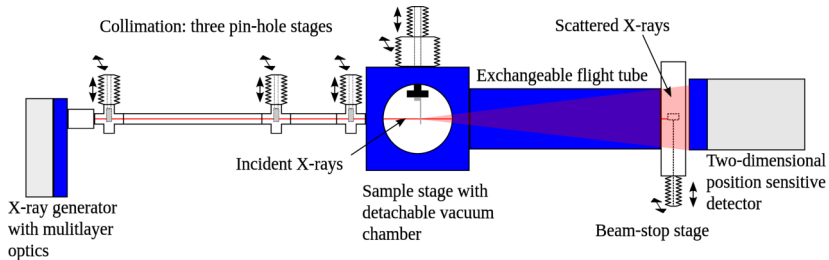
Particulate systems

- Size distribution of SiO₂ nanoparticles

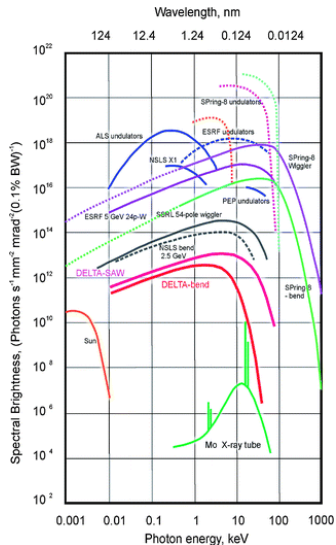
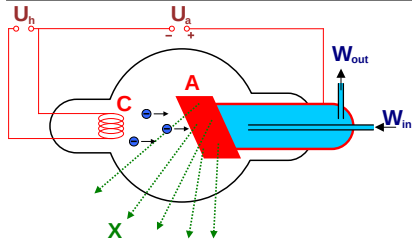
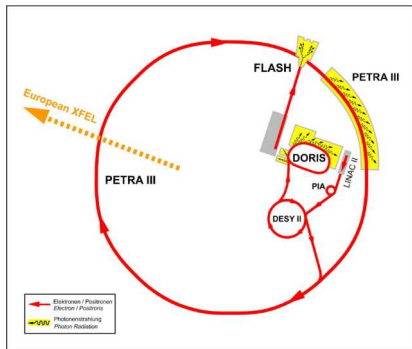
- Proteins – biological macromolecules

Summary

Pinhole camera

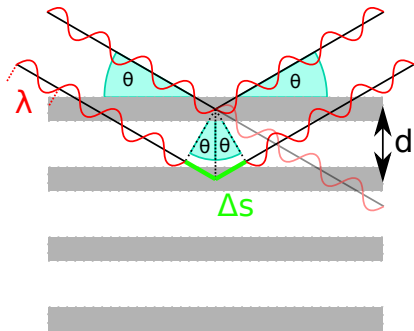


X-ray tube and synchrotron

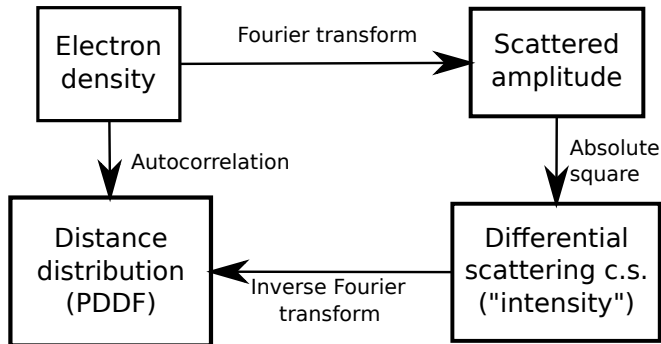


Bragg's law

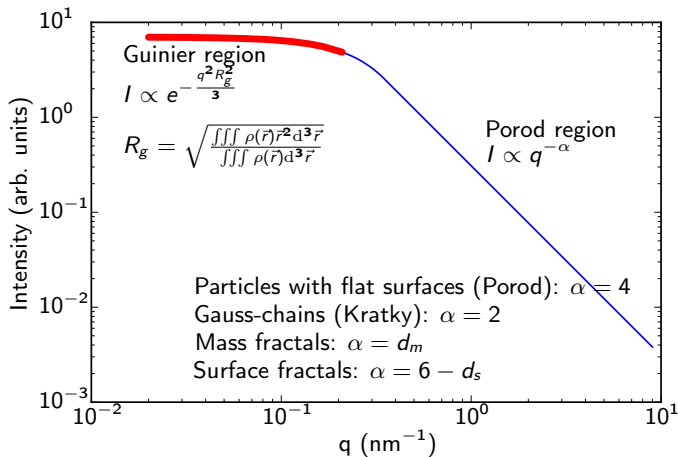
- ▶ Periodic sample (d repeat distance)
- ▶ θ incidence and reflection angle
- ▶ Constructive interference at the detector: waves reflected from neighbouring planes meet *in phase*
- ▶ $\Delta s = n\lambda$ where $n \in \mathbb{N}$
- ▶ From geometry: $\Delta s = 2d \sin \theta$
- ▶ $2d \sin \theta = n\lambda$
- ▶ $\frac{4\pi}{\lambda} \sin \theta = \frac{2\pi}{d} n$
- ▶ $q = \frac{2\pi}{d} n$



Connection between structure and scattering



Guinier and Porod limits



Outline

Recapitulation

Self-assembled systems

Ordered phases

Multilamellar and hexagonal phases

Unilamellar vesicles

Micellar systems

Continuous – hierarchical – systems

Anisotropy and porosity of activated carbons

In situ experiments on a gold-cysteine self-assembling nanocomplex

Particulate systems

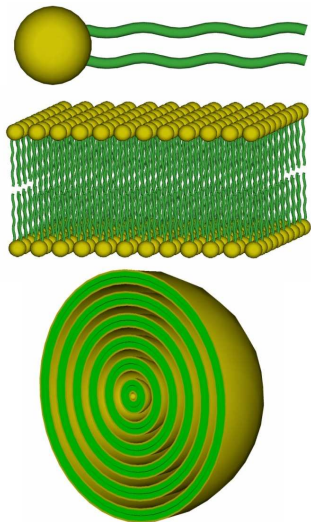
Size distribution of SiO_2 nanoparticles

Proteins – biological macromolecules

Summary

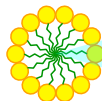
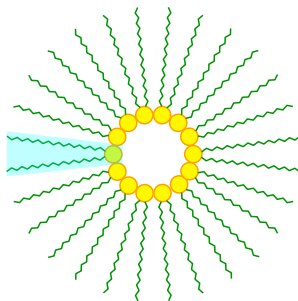
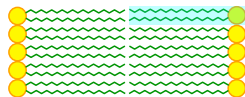
Lipid systems, liposomes

- ▶ Amphipatic molecules: hydrophilic headgroups, hydrophobic carbon chains
- ▶ Self-assemble in aqueous solution
- ▶ Cell membranes of living organisms
- ▶ Other similar molecules: surfactants, detergents etc.
- ▶ Application in research & industry:
 - ▶ Model membranes
 - ▶ Drug carrier vehicles
 - ▶ Nanoreactors
 - ▶ ...
- ▶ Phase transitions
 - ▶ Thermotropic
 - ▶ Lyotropic



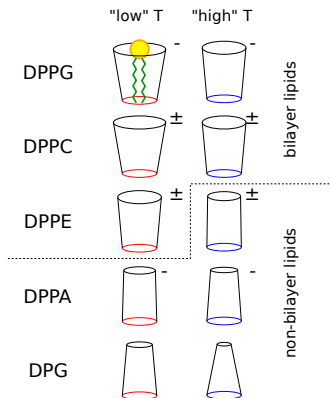
Self-assembled structures of phospholipid systems

- ▶ The self-assembled structure is determined by:
 - ▶ Shape of the lipid molecule
 - ▶ Length and flexibility of the carbon chains
 - ▶ Electrostatic charge of the headgroups
- ▶ Bilayer lipids: approximately cylindrical
- ▶ Non-bilayer lipids: conical shape
 - ▶ Large headgroup cross-section area: micelle / hexagonal phase
 - ▶ Small headgroup cross-section area: inverse micelle / inverse hexagonal phase



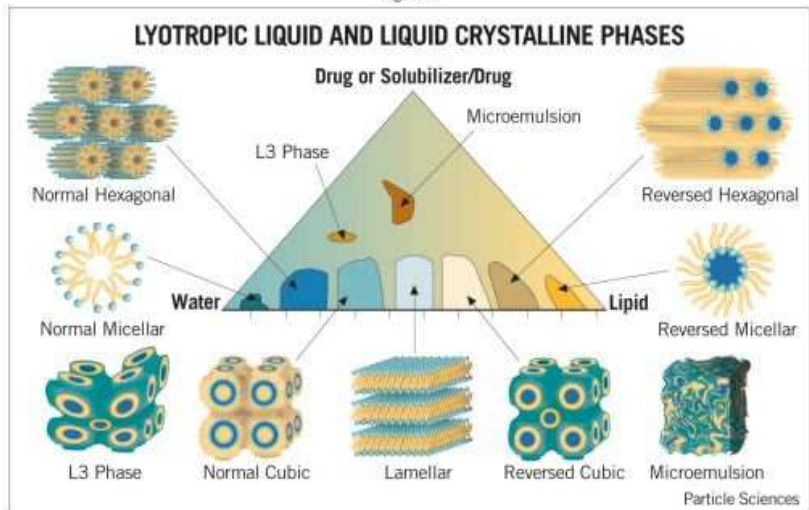
Self-assembled structures of phospholipid systems

- ▶ The self-assembled structure is determined by:
 - ▶ Shape of the lipid molecule
 - ▶ Length and flexibility of the carbon chains
 - ▶ Electrostatic charge of the headgroups
- ▶ Bilayer lipids: approximately cylindrical
- ▶ Non-bilayer lipids: conical shape
 - ▶ Large headgroup cross-section area: micelle / hexagonal phase
 - ▶ Small headgroup cross-section area: inverse micelle / inverse hexagonal phase

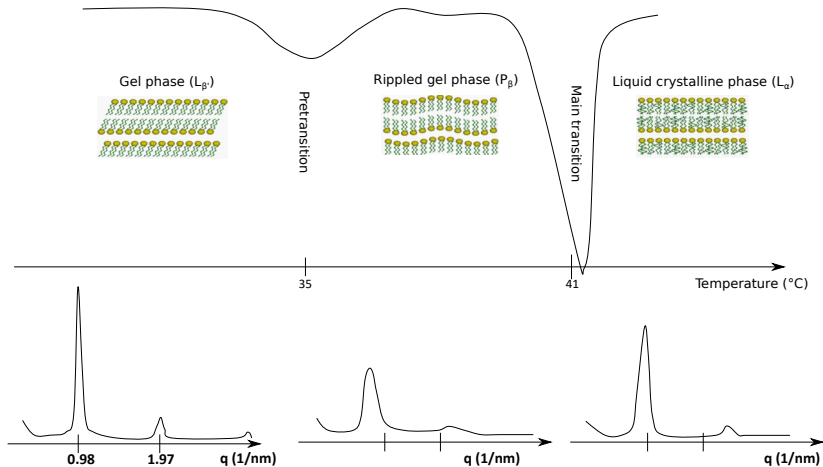


Lyotropic phases of lipid/water systems

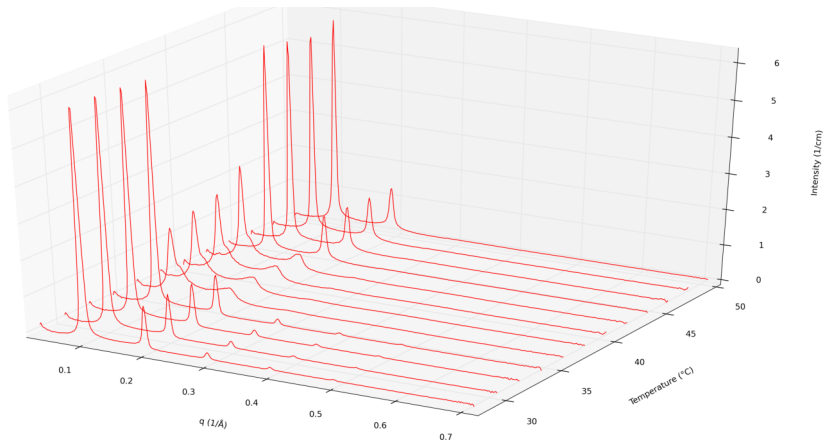
Figure 1



Thermotropic phases of DPPC/water mixtures

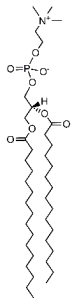
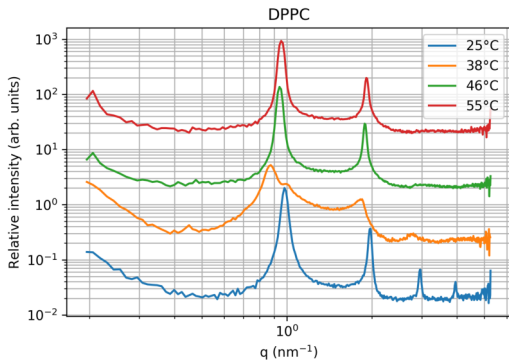


Thermotropic phases of DPPC: SAXS



Relative peak positions: 1, 2, 3, 4 \rightarrow lamellar

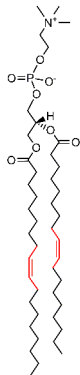
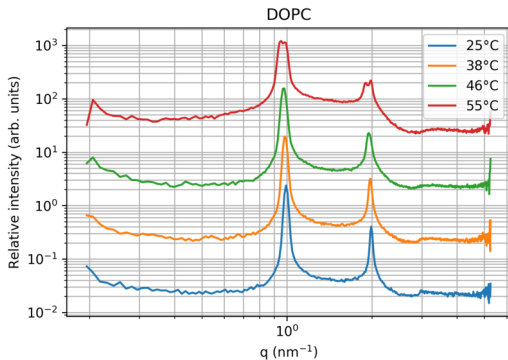
Thermotropic phases of DPPC: SAXS



Relative peak positions: 1, 2, 3, 4 → lamellar

Temperature	25°C	38°C	46°C	55°C
Phase	L _β	P _{β'}	L _α	L _α
Repeat distance	6.373 nm	"7.193 nm*"	6.657 nm	6.569 nm

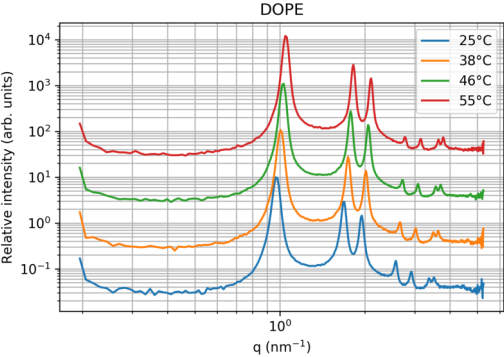
DOPC



Relative peak positions: 1, 2, (3)

Temperature	25°C	38°C	46°C	55°C
Phase	L_{α}	L_{α}	L_{α}	L_{α}
Repeat distance	6.323 nm	6.370 nm	6.440 nm	6.642 nm 6.335 nm

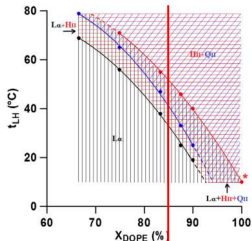
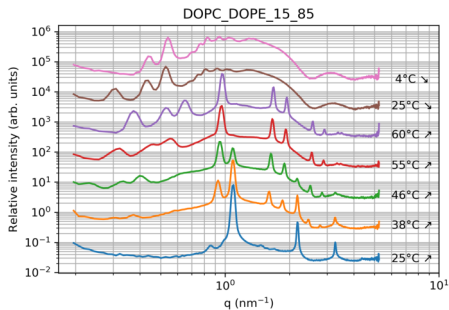
DOPE: hexagonal phase



Relative peak positions: 1, $\sqrt{3}$, 2, $\sqrt{7}$, 3, $\sqrt{12}$, $\sqrt{13}$

Temperature	25°C	38°C	46°C	55°C
Phase	H _{II}	H _{II}	H _{II}	H _{II}
Lattice parameter	6.458 nm	6.244 nm	6.119 nm	5.989 nm

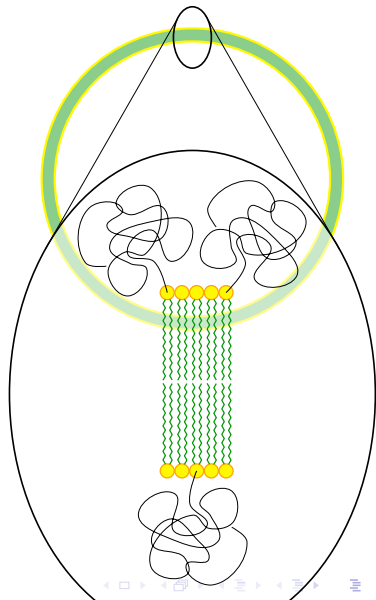
Coexistence of phases



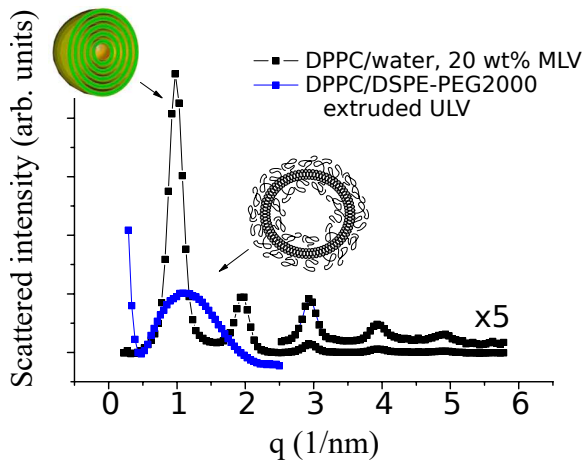
- ▶ Room temperature: lamellar phase (L_{α})
- ▶ 38 °C: appearance of the inverse hexagonal phase (H_{II})
- ▶ 46 °C: the cubic phase (Q_{II}) appears, three phases coexist
- ▶ 55 °C: the lamellar phase vanishes
- ▶ after cooling: the cubic phase remains, the lamellar phase is not recovered: memory effect!

Sterically stabilized unilamellar vesicles

- ▶ Unilamellar vesicle: a single phospholipid bilayer
- ▶ Hydration of lipids: multilamellar vesicles are formed *spontaneously*
- ▶ “Unilamellarization”: ultrasound treatment / extrusion
- ▶ Avoiding spontaneous fusion to multilamellar vesicles:
 - ▶ Charged lipids
 - ▶ Sterical stabilization: e.g. with PEG-conjugated lipids
- ▶ Primary application: drug carrier and targeting agents ⇒ size is critical!

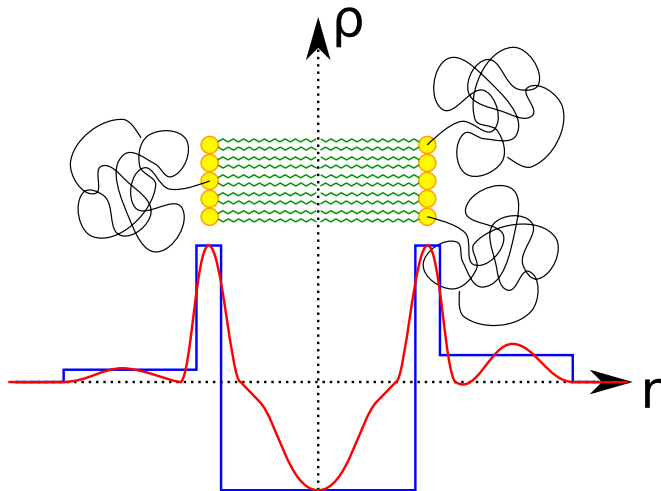


Sterically stabilized vesicles



- ▶ Less electrons in the object \Rightarrow weaker scattering
- ▶ No layer-layer correlation \Rightarrow no peaks
- ▶ What we see is the *phospholipid bilayer form factor*

Scattering of a phospholipid bilayer



$$I_{SSL}(q) = [F_{PEG,in}(q) + F_{head,in}(q) + F_{CH}(q) + F_{head,out}(q) + F_{PEG,out}(q)]^2$$

Scattering of a bilayer

$$I_{\text{SSL}}(q) = [F_{\text{PEG,in}}(q) + F_{\text{head,in}}(q) + F_{\text{CH}}(q) + F_{\text{head,out}}(q) + F_{\text{PEG,out}}(q)]^2$$

- ▶ Every term is a **step function** or a **Gaussian curve**

$$\rho(q) = \begin{cases} \rho_0 & \text{if } |r - r_0| < \sigma \\ 0 & \text{otherwise} \end{cases} \quad \rho(q) = \frac{\rho_0}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r-r_0)^2}{2\sigma^2}}$$

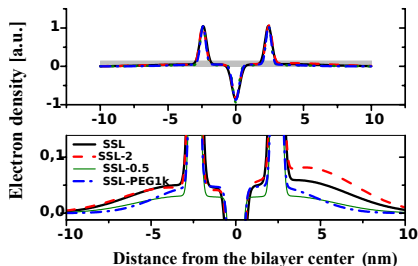
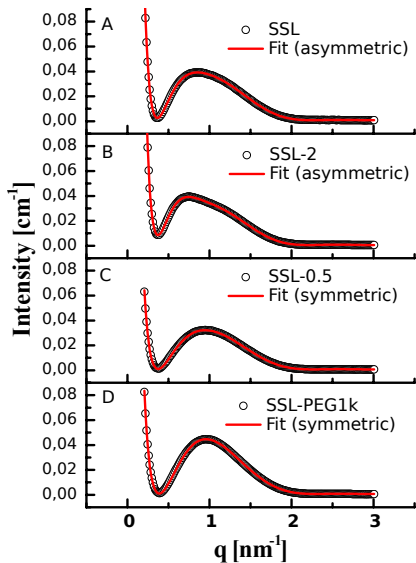
- ▶ Model parameters:

	ρ	r	σ
Inner PEG	$\rho_{\text{PEG,in}}$	$r_{\text{PEG,in}}$	$\sigma_{\text{PEG,in}}$
Inner headgroup	ρ_{head}	$-r_{\text{head}}$	σ_{head}
Carbon chain	-1	0	σ_{tail}
Outer headgroup	ρ_{head}	r_{head}	σ_{head}
Outer PEG	$\rho_{\text{PEG,out}}$	$r_{\text{PEG,out}}$	$\sigma_{\text{PEG,out}}$

+ global intensity scaling factor (A) + constant background (C) + mean vesicle radius (R_0) + spread of the vesicle radius (δR)

- ▶ Asymmetric model (PEGs are different): 14 parameters
- ▶ Symmetric model (PEGs are equivalent): 11 parameters

Sterically stabilized vesicles



SSL: HSPC +
DSPE-PEG2000

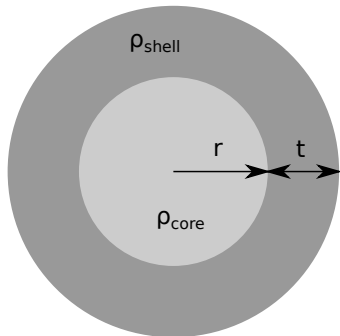
SSL-2: HSPC +
2×DSPE-PEG2000

SSL-0.5: HSPC +
0.5×DSPE-PEG2000

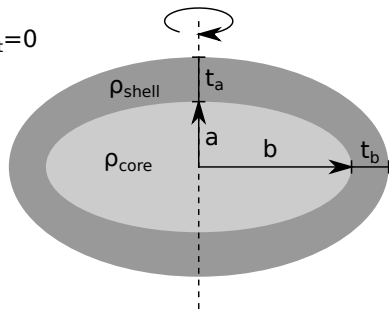
SSL-PEG1k: HSPC +
DSPE-PEG1000

Micelles

- ▶ Self-assembling systems composed of amphipatic molecules
- ▶ Conical shape: large hydrophilic head, narrow hydrophobic tail
- ▶ Critical micelle concentration (CMC)
- ▶ Not only spherical (even when only one component!)



$$\rho_{\text{solvent}}=0$$



Bicelles

- ▶ Two components: long-chained bilayer lipid and short-chained detergent

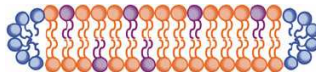
- ▶ The shape is controlled by:

$$q = C_{\text{lipid}} / C_{\text{detergents}}$$

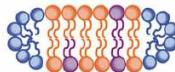
- ▶ $q = 0$: detergent micelle
- ▶ $q \rightarrow \infty$: bilayer
- ▶ Importance: small carriers for membrane proteins

- ▶ Typical example: DHPC-DMPC bicelle

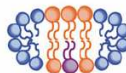
- ▶ DHPC: 1,2-Dihexanoyl-sn-Glycero-3-Phosphocholine
- ▶ DMPC: 1,2-Dimyristoyl-sn-Glycero-3-Phosphocholine



$q = 2.5$



$q = 1$



$q = 0.5$



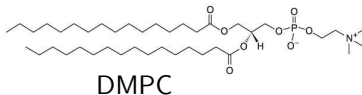
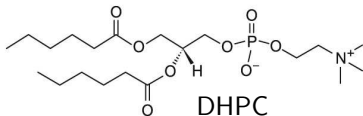
$q \sim 0$



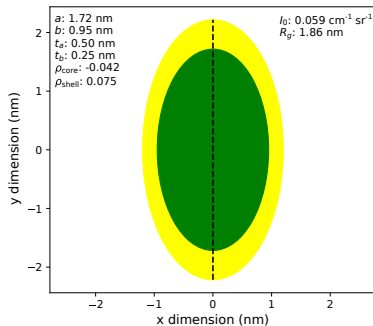
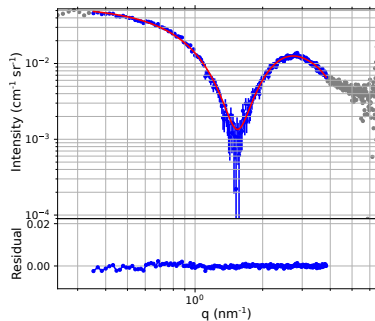
Detergent



Long chain lipid

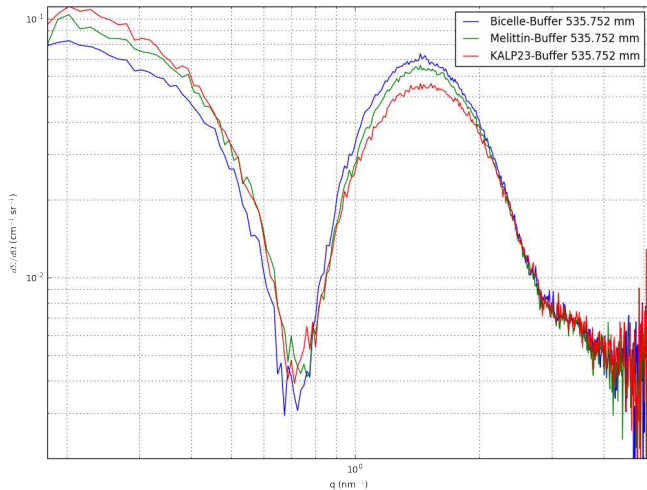


Scattering of a DHPC micelle



- ▶ Scattering: similar to the lipid bilayers
- ▶ Guinier region
- ▶ Fitting: micelle shape

Peptide-carrying DHPC-DMPC bicelles



Outline

Recapitulation

Self-assembled systems

- Ordered phases

 - Multilamellar and hexagonal phases

- Unilamellar vesicles

- Micellar systems

Continuous – hierarchical – systems

- Anisotropy and porosity of activated carbons

- In situ experiments on a gold-cysteine self-assembling nanocomplex

Particulate systems

- Size distribution of SiO₂ nanoparticles

- Proteins – biological macromolecules

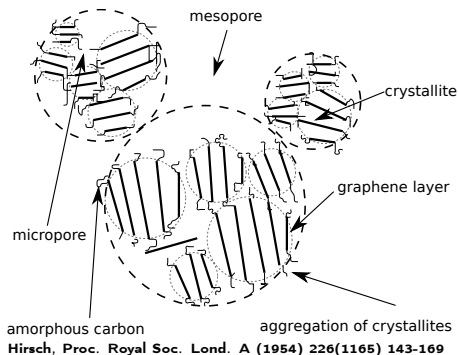
Summary

Activated carbons

Activated carbons

- ▶ Adsorbent, substrate, structural material
- ▶ Hierarchical structure
- ▶ Preparation:
 1. Pyrolysis: organic \rightarrow C
 2. Activation: pore formation
- ▶ Tailorable
 - ▶ choice of the precursor
 - ▶ parameters of the activation
- ▶ Anisotropy: not utilized (but could be...)

Model of the hierarchical structure

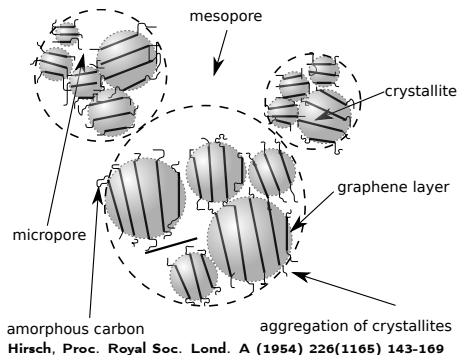


Activated carbons

Activated carbons

- ▶ Adsorbent, substrate, structural material
- ▶ Hierarchical structure
- ▶ Preparation:
 1. Pyrolysis: organic \rightarrow C
 2. Activation: pore formation
- ▶ Tailorable
 - ▶ choice of the precursor
 - ▶ parameters of the activation
- ▶ Anisotropy: not utilized (but could be...)

Model of the hierarchical structure

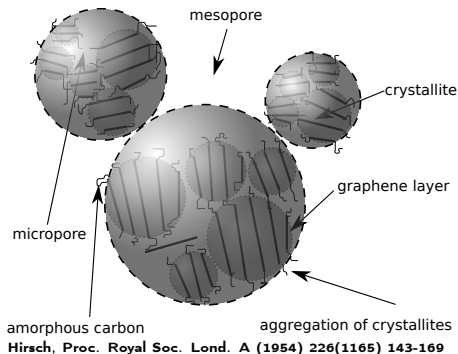


Activated carbons

Activated carbons

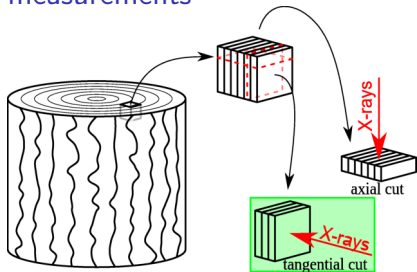
- ▶ Adsorbent, substrate, structural material
- ▶ Hierarchical structure
- ▶ Preparation:
 1. Pyrolysis: organic \rightarrow C
 2. Activation: pore formation
- ▶ Tailorable
 - ▶ choice of the precursor
 - ▶ parameters of the activation
- ▶ Anisotropy: not utilized (but could be...)

Model of the hierarchical structure



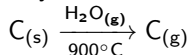
Sample preparation

Sample preparation for SAXS measurements



- ▶ Pyrolysis of 1 cm³ wooden cubes (700 °C) → 6 × 6 × 6 mm³ carbon cubes

- ▶ Physical activation:



- ▶ Mass decrease (conversion) with increasing activation time:

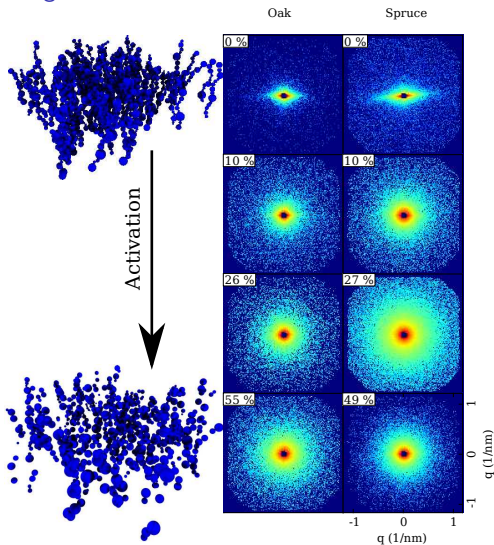
	Fagus sylvatica (beech)	Quercus robur (oak)	Picea abies (spruce)
0 min	0 %	0 %	0 %
15 min	9 %	10 %	10 %
45 min	26 %	26 %	27 %
90 min	54 %	55 %	49 %

- ▶ SAXS measurements: synchrotron beamlines (Hamburg, Berlin)

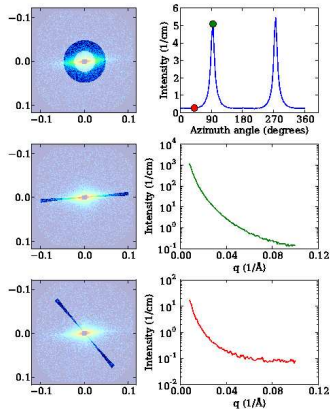
For details, see: Wacha, Varga, Vainio, Hoell, Bóta (2011) Carbon 49(12) 3958-3971.

SAXS on activated carbons

Tangential cut

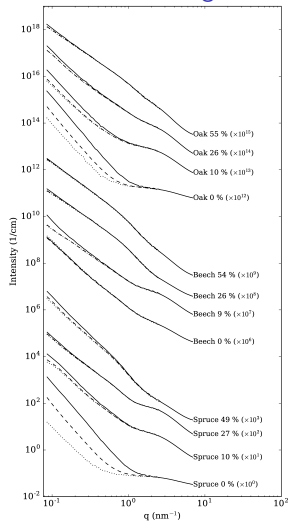


- ▶ Horizontal scattering pattern: vertical fibrils
- ▶ Decrease in anisotropy: breaking of the fibrils, pore formation
- ▶ Characterization of anisotropy: azimuthal scattering curves, sector averaging



Extent of the anisotropy in real space

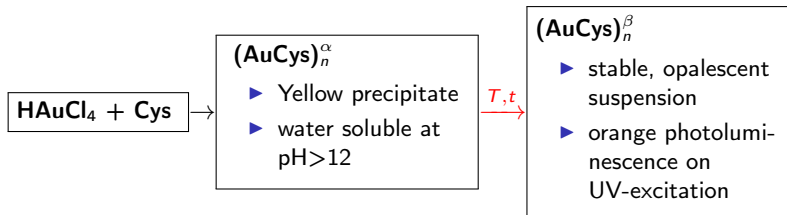
Radial sector averages



- ▶ Radial scattering curves from the scattering patterns
 - ▶ Averaging over the full 2π (---)
 - ▶ Narrowed to the region of the most intensive azimuthal peak (—)
 - ▶ Perpendicular to the previous direction (···)
- ▶ Anisotropy does not appear at small sizes ($q > 2 \text{ nm}^{-1} \rightarrow d < 1.5 \text{ nm}$)
- ▶ Anisotropy decreases with activation
- ▶ Power-law functions (\rightarrow fractal dimension) and Guinier regions (\rightarrow radius of gyration)
- ▶ Two Guinier regions
 - ▶ Small conversion (short activation time): micropores
 - ▶ Large conversion (long activation time): mesopores
- ▶ Mass fractal \rightarrow surface fractal transition
 - ▶ Spruce: surface fractal appears after 49 % burn-off: microcracks
 - ▶ Beech: no surface fractal: inherently porous?

Photoluminescent gold-cystein nanocomplexes

- ▶ Protein-stabilized supramolecular gold clusters: photoluminescence
- ▶ Au-Cys nanocomplex: a simple model for uncovering the stabilizing mechanism

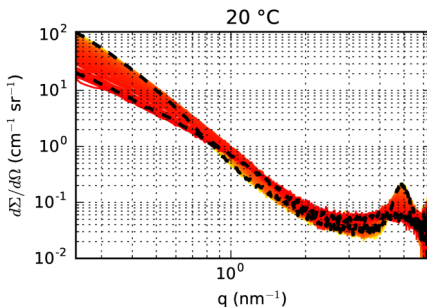
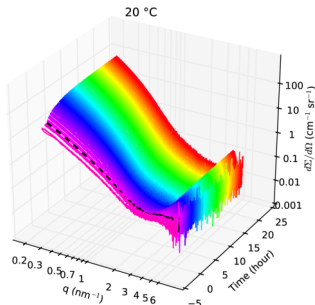


- ▶ The speed of transition strongly depends on the temperature of incubation, ranging from a few hours to a day \Rightarrow **time-resolved SAXS on CREDO**

Söptei *et. al.* 2015 Coll.Surf.A 470, 8-14.

TR SAXS on the Au-Cys nanocomplex

Incubation at 20 °C



Curvature at small $q \rightarrow$ Guinier

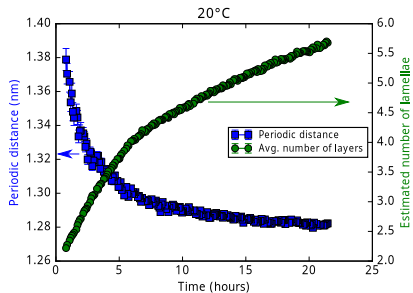
- ▶ Objects with well-defined sizes
- ▶ Moves left \rightarrow increase in size
- ▶ Increasing intensity \rightarrow their number increases
- ▶ Starts with $I \propto q^{-2} \rightarrow$ thin lamellae (generalized Guinier)

Peak at the high- q limit

- ▶ Periodic structure
- ▶ Increasing intensity \rightarrow more perfect periodicity

Automated model fitting

Number of layers and periodicity



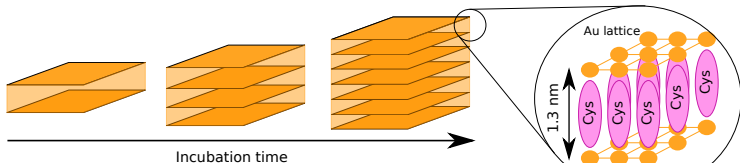
- ▶ Guinier approximation for extended lamellae:

$$I_{\text{thickness}} \approx G \cdot q^{-2} e^{-q^2 R_T^2} \rightarrow$$

thickness of the homogeneous lamella:

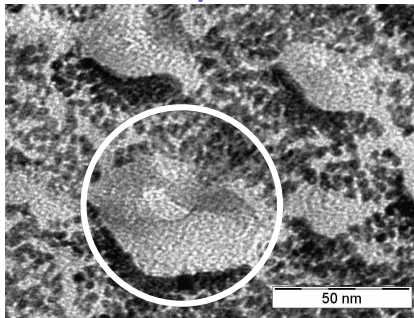
$$T = \sqrt{12} R_T$$

- ▶ Final periodic distance: 1.29 nm
- ▶ Fine structure of the lamellae: Au layers above each other with ≈ 1.3 nm distance, the Cys molecules acting as spacers
- ▶ Well-correlated with the increase of photoluminescence intensity: 0.9208



Automated model fitting

Lamellae as seen by TEM



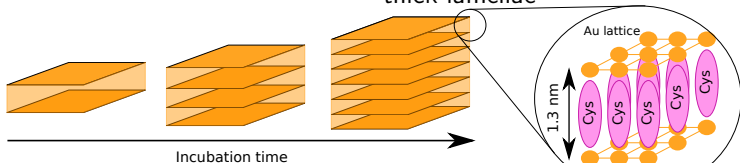
- ▶ Guinier approximation for extended lamellae:

$$I_{\text{thickness}} \approx G \cdot q^{-2} e^{-q^2 R_T^2} \rightarrow$$

thickness of the homogeneous lamella:

$$T = \sqrt{12} R_T$$

- ▶ Final periodic distance: 1.29 nm
- ▶ Fine structure of the lamellae: Au layers above each other with ≈ 1.3 nm distance, the Cys molecules acting as spacers
- ▶ Well-correlated with the increase of photoluminescence intensity: 0.9208
- ▶ FF-TEM measurements: a few nm thick lamellae



Outline

Recapitulation

Self-assembled systems

- Ordered phases

 - Multilamellar and hexagonal phases

- Unilamellar vesicles

- Micellar systems

Continuous – hierarchical – systems

- Anisotropy and porosity of activated carbons

- In situ experiments on a gold-cysteine self-assembling nanocomplex

Particulate systems

- Size distribution of SiO_2 nanoparticles

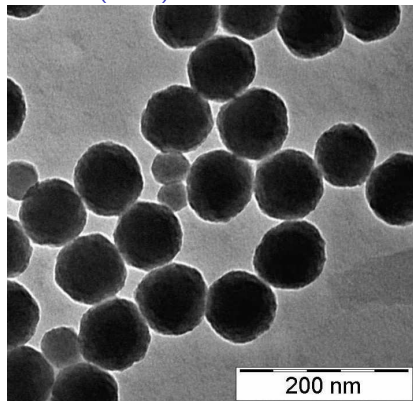
- Proteins – biological macromolecules

Summary

Size distribution of SiO₂ nanoparticles

Institute for Reference Materials and Measurements, Joint Research Centre of the European Commission: introducing a new SiO₂ particle size standard. Certification of the new material with several SAXS instruments

ERM FD-101b: candidate reference material (CRM)



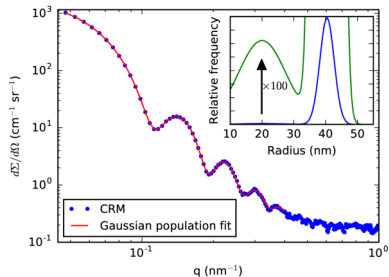
► Methods of size determination:

1. Guinier fit: $I(q \ll 1/R) \approx I_0 e^{-\frac{q^2 R^2}{5}}$
2. Fitting of the sphere form factor:
 $I(q) = \Phi_{\text{sphere}}(q, R) \equiv V_R^2 \left[\frac{3}{(qR)^3} (\sin(qR) - qR \cos(qR)) \right]^2$
3. Fitting of a sphere distribution:
 $I(q) = \int_0^{\infty} p(R) \Phi_{\text{sphere}}(q, R) dR$
4. Monte Carlo method: R_i population with w_i statistical weights where $|I(q) - \sum_i w_i \Phi_{\text{sphere}}(q, R_i)|$ is minimized

Size distribution of SiO₂ nanoparticles

Institute for Reference Materials and Measurements, Joint Research Centre of the European Commission: introducing a new SiO₂ particle size standard. Certification of the new material with several SAXS instruments

ERM FD-101b: candidate reference material (CRM)



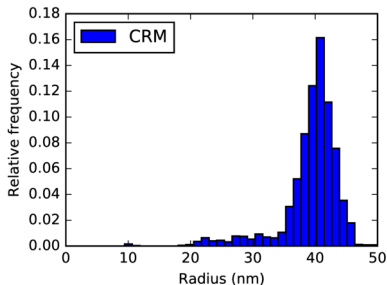
► Methods of size determination:

1. Guinier fit: $I(q \ll 1/R) \approx I_0 e^{-\frac{q^2 R^2}{5}}$
2. Fitting of the sphere form factor:
 $I(q) = \Phi_{\text{sphere}}(q, R) \equiv V_R^2 \left[\frac{3}{(qR)^3} (\sin(qR) - qR \cos(qR)) \right]^2$
3. Fitting of a sphere distribution:
 $I(q) = \int_0^{\infty} p(R) \Phi_{\text{sphere}}(q, R) dR$
4. Monte Carlo method: R_i population with w_i statistical weights where $|I(q) - \sum_i w_i \Phi_{\text{sphere}}(q, R_i)|$ is minimized

Size distribution of SiO₂ nanoparticles

Institute for Reference Materials and Measurements, Joint Research Centre of the European Commission: introducing a new SiO₂ particle size standard. Certification of the new material with several SAXS instruments

Monte Carlo size determination



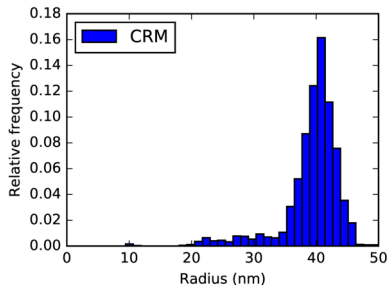
► Methods of size determination:

1. Guinier fit: $I(q \ll 1/R) \approx I_0 e^{-\frac{q^2 R^2}{5}}$
2. Fitting of the sphere form factor:
 $I(q) = \Phi_{\text{sphere}}(q, R) \equiv V_R^2 \left[\frac{3}{(qR)^3} (\sin(qR) - qR \cos(qR)) \right]^2$
3. Fitting of a sphere distribution:
 $I(q) = \int_0^{\infty} p(R) \Phi_{\text{sphere}}(q, R) dR$
4. Monte Carlo method: R_i population with w_i statistical weights where $|I(q) - \sum_i w_i \Phi_{\text{sphere}}(q, R_i)|$ is minimized

Size distribution of SiO₂ nanoparticles

Institute for Reference Materials and Measurements, Joint Research Centre of the European Commission: introducing a new SiO₂ particle size standard. Certification of the new material with several SAXS instruments

Monte Carlo size determination



► Methods of size determination:

1. Guinier fit: $I(q \ll 1/R) \approx I_0 e^{-\frac{q^2 R^2}{5}}$
2. Fitting of the sphere form factor:
 $I(q) = \Phi_{\text{sphere}}(q, R) \equiv V_R^2 \left[\frac{3}{(qR)^3} (\sin(qR) - qR \cos(qR)) \right]^2$
3. Fitting of a sphere distribution:
 $I(q) = \int_0^{\infty} p(R) \Phi_{\text{sphere}}(q, R) dR$
4. Monte Carlo method: R_i population with w_i statistical weights where $|I(q) - \sum_i w_i \Phi_{\text{sphere}}(q, R_i)|$ is minimized

A favourable side-effect: CREDO has been certified by IRMM for nanoparticle size distribution determination

Biological Small-Angle X-ray Scattering

BioSAXS

- ▶ Biorelevant macromolecules
- ▶ Mainly size- and shape determination assuming particles of homogeneous electron density
- ▶ Key parameters: R_g , $I_0 \equiv \lim_{q \rightarrow 0} I(q)$
- ▶ Information to be obtained:
 - ▶ Size, (low resolution) shape, volume and molecular mass of the protein
 - ▶ Flexibility/folding state (folded/disordered)
 - ▶ Validation of crystal structures
 - ▶ Aligning the relative positions of known domains

Drawbacks / caveats

- ▶ Low scattering contrast \Rightarrow bad signal/noise ratio
- ▶ Dilute sample (otherwise Guinier approximation breaks down)
- ▶ Purified sample (esp. contaminating large molecules)
- ▶ Monodisperse sample (avoid oligomerization, aggregation)
- ▶ Featureless scattering curve: danger of “overfitting”
- ▶ Uncertainties of background subtraction (solvent scattering)
- ▶ Phase problem \Rightarrow the uniqueness of the determined shape

The BioSAXS method

- ▶ Well-established and validated algorithms and methods available
- ▶ Basic assumptions: the protein solution is a *monodisperse* population of *independent, homogeneous* nanoparticles
 - independent:** no interparticle interference, Guinier approximation holds
 - monodisperse:** no oligomerization, no aggregation
 - homogeneous:** simple shape fitting; SAXS is blind on the atomic length-scale!



Interpretation of BioSAXS measurements

- ▶ Guinier approximation: $I(q \ll R_g) \propto I_0 e^{-\frac{q^2 R_g^2}{3}}$; $I_0 = (\Delta\rho)^2 V^2$.
- ▶ Porod invariant: $Q \equiv \frac{1}{2\pi^2} \int_0^\infty q^2 I(q) dq = 2\pi^2 (\Delta\rho)^2 V$
- ▶ Porod volume: $V_{\text{Porod}} = 2\pi^2 I_0 / Q$
- ▶ First steps:

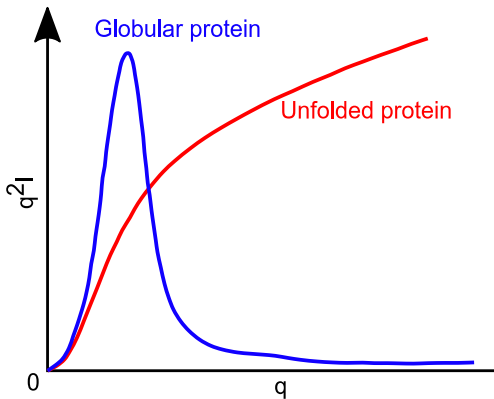
1. Subtraction of the solvent background (corrected by the volume fraction of the protein)
2. Guinier fit $\rightarrow I_0, R_g$
3. Porod invariant $\rightarrow V_{\text{Porod}}$
4. Inverse Fourier: $I(q) \rightarrow p(r)$ pair distance distribution function (PDDF)
5. I_0, R_g can be obtained from $p(r)$:

$$I_0 = \int_0^\infty p(r) dr; \quad R_g^2 = \frac{\int_0^\infty p(r) r^2 dr}{2 \int_0^\infty p(r) dr}$$

6. Compare the I_0 and R_g obtained from the two methods
 7. Further interpretation...
- ▶ ATSAS: software suite for BioSAXS data processing and interpretation (EMBL Hamburg, Research Group of Dmitri Svergun)

The Kratky plot

- ▶ High- q part of the scattering of a polymer chain following Gaussian statistics: $I(q \rightarrow \infty) \propto \frac{2}{q^2 R_g^2}$
- ▶ Kratky plot: $q^2 I - q$. Behaviour in the $q \rightarrow \infty$ limit:
 - ▶ **Folded proteins** ($I \propto q^{-4}$): tends to 0
 - ▶ **Disordered proteins** ($I \propto q^{-2}$): constant or divergent



Source: <https://www-ssrl.slac.stanford.edu/~saxs/analysis/assessment.htm>

Protein shape fitting from small-angle scattering

Fitting of geometrical shapes to scattering curves or PDDFs

- ▶ BODIES program (part of ATSAS)
- ▶ Ball, hollow sphere, ellipsoid, dumbbell etc.
- ▶ Very few parameters

Dummy atom model (DAM)

- ▶ Constructing the shape from tightly packed (fcc or hcp lattice) spherical building blocks
- ▶ Monte Carlo algorithm
 1. Random configuration
 2. Small, random modification of the configuration (add/remove a unit)
 3. Calculate scattering
 4. Compare the measured and calculated scattering
 - ▶ Better fit: keep the change
 - ▶ Worse fit: drop the change (or keep it with a low probability)
 5. Repeat from step #2 until needed
- ▶ Many parameters: possible ambiguity of the results

Protein shape fitting from small-angle scattering

Fitting of geometrical shapes to scattering curves or PDDFs

- ▶ BODIES program (part of ATSAS)
- ▶ Ball, hollow sphere, ellipsoid, dumbbell etc.
- ▶ Very few parameters

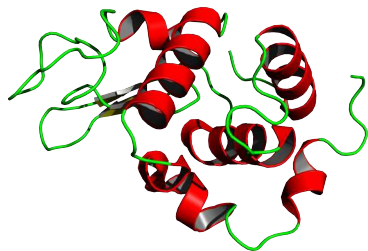
Dummy atom model (DAM)

- ▶ Constructing the shape from tightly packed (fcc or hcp lattice) spherical building blocks
- ▶ Monte Carlo algorithm
 1. Random configuration
 2. Small, random modification of the configuration (add/remove a unit)
 3. Calculate scattering
 4. Compare the measured and calculated scattering
 - ▶ Better fit: keep the change
 - ▶ Worse fit: drop the change (or keep it with a low probability)
 5. Repeat from step #2 until needed
- ▶ Many parameters: possible ambiguity of the results

Lysozyme – A “typical” BioSAXS experiment

Crystal structure

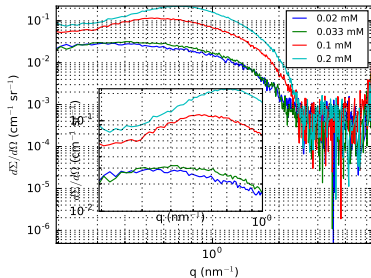
- ▶ Well-known protein (“veterinary horse”)



Lysozyme – A “typical” BioSAXS experiment

Effect of concentration

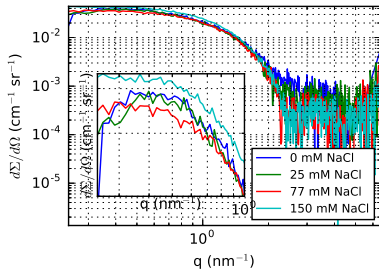
Lizozim



- ▶ Well-known protein (“veterinary horse”)
- ▶ Correlation peak
 - ▶ Caused by el.stat. repulsion
 - ▶ Radius of gyration cannot be determined
 - ▶ How to get rid of it?
 - ▶ Dilution

Lysozyme – A “typical” BioSAXS experiment

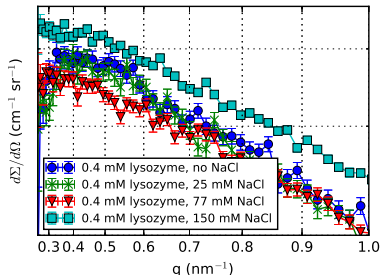
Effect of ionic strength



- ▶ Well-known protein (“veterinary horse”)
- ▶ Correlation peak
 - ▶ Caused by el.stat. repulsion
 - ▶ Radius of gyration cannot be determined
 - ▶ How to get rid of it?
 - ▶ Dilution
 - ▶ Salting (screening the repulsion)

Lysozyme – A “typical” BioSAXS experiment

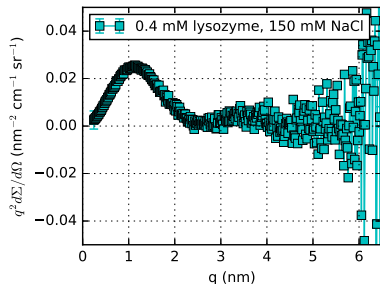
Guinier plot



- ▶ Well-known protein (“veterinary horse”)
- ▶ Correlation peak
 - ▶ Caused by el.stat. repulsion
 - ▶ Radius of gyration cannot be determined
 - ▶ How to get rid of it?
 - ▶ Dilution
 - ▶ Salting (screening the repulsion)
- ▶ Guinier plot ($\log I$ vs. q^2): assessing the $I \propto \exp(-q^2 R_g^2/3)$ shape

Lysozyme – A “typical” BioSAXS experiment

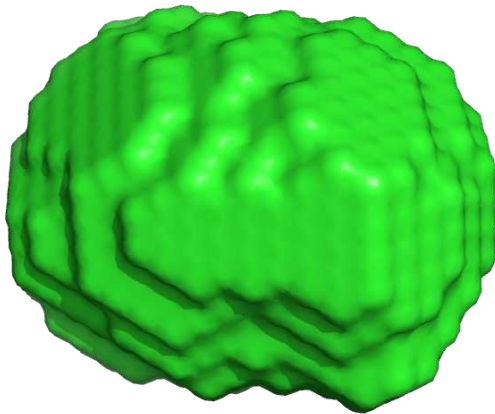
Kratky plot



- ▶ Well-known protein (“veterinary horse”)
- ▶ Correlation peak
 - ▶ Caused by el.stat. repulsion
 - ▶ Radius of gyration cannot be determined
 - ▶ How to get rid of it?
 - ▶ Dilution
 - ▶ Salting (screening the repulsion)
- ▶ Guinier plot ($\log I$ vs. q^2): assessing the $I \propto \exp(-q^2 R_g^2/3)$ shape
- ▶ Kratky plot ($q^2 I$ vs. q): folded protein

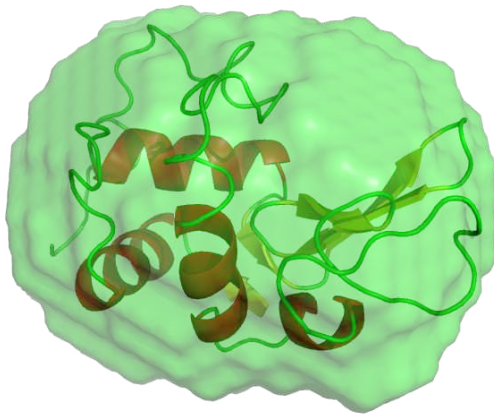
The shape of lysozyme

„Dummy atom model” – coarse-grained description



The shape of lysozyme

„Dummy atom model” – coarse-grained description

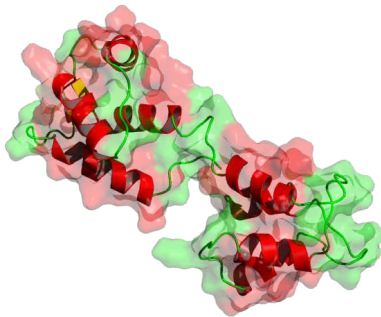


Good agreement with the crystal structure!

Calmodulin

- ▶ Highly abundant plasma protein of eukaryotic cells ($\approx 1\%$)
- ▶ Key element of Ca^{2+} -induced signal pathways

Apo (Ca^{2+} -free) conformation (MX)

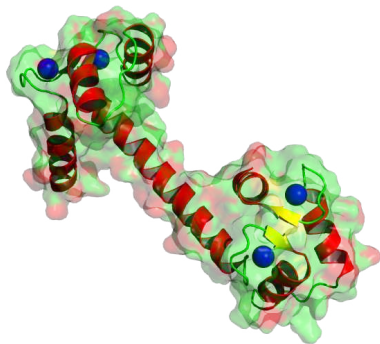


Envelope: Van der Waals surface

Calmodulin

- ▶ Highly abundant plasma protein of eukaryotic cells ($\approx 1\%$)
- ▶ Key element of Ca^{2+} -induced signal pathways
- ▶ Changes shape on Ca^{2+} binding
 - ▶ The “EF-hand” motifs open in both end-domains: hydrophobic pockets open up
 - ▶ End domains are displaced
 - ▶ Secondary structure of the linker part: loop \rightarrow helix (known crystallization artefact!)

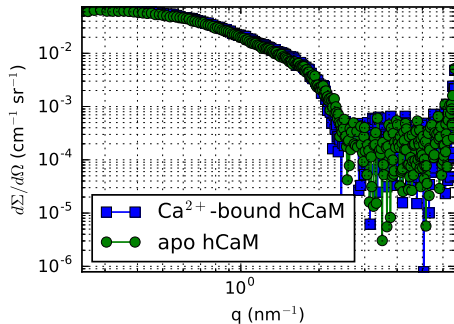
Ca^{2+} -bound conformation (MX)



Envelope: Van der Waals surface

Calmodulin – SAXS results

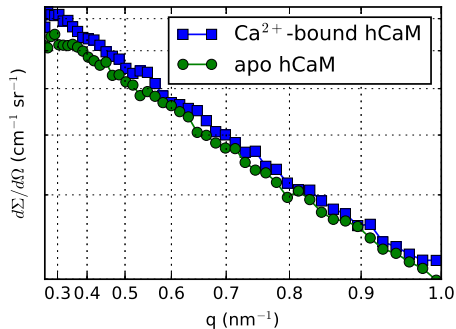
Scattering curves



- ▶ Very similar scattering curves
- ▶ Scattering curves: dumbbell shape

Calmodulin – SAXS results

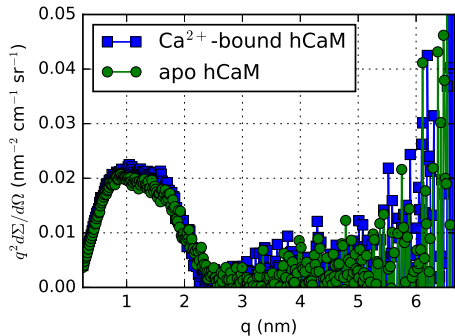
Guinier plot



- ▶ Very similar scattering curves
- ▶ Scattering curves: dumbbell shape
- ▶ Similar radii of gyration

Calmodulin – SAXS results

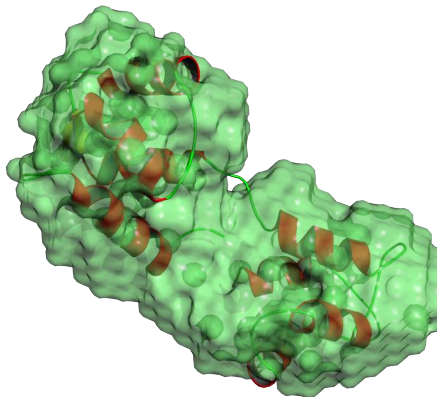
Kratky plot



- ▶ Very similar scattering curves
- ▶ Scattering curves: dumbbell shape
- ▶ Similar radii of gyration
- ▶ Partially disordered (linker part?)

Calmodulin – SAXS results

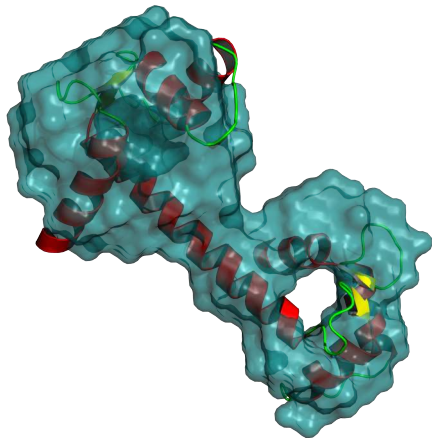
DAM: apo conformation



- ▶ Very similar scattering curves
- ▶ Scattering curves: dumbbell shape
- ▶ Similar radii of gyration
- ▶ Partially disordered (linker part?)
- ▶ Dummy atom model:
 - ▶ Dumbbell shape
 - ▶ Apo conformation more "loose"

Calmodulin – SAXS results

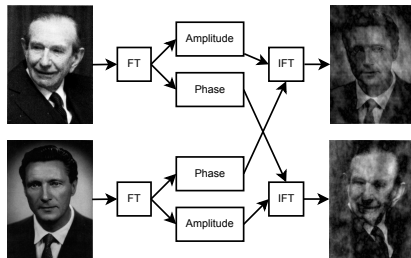
DAM: Ca^{2+} -bound conformation



- ▶ Very similar scattering curves
- ▶ Scattering curves: dumbbell shape
- ▶ Similar radii of gyration
- ▶ Partially disordered (linker part?)
- ▶ Dummy atom model:
 - ▶ Dumbbell shape
 - ▶ Apo conformation more “loose”
 - ▶ Ca^{2+} binding makes the structure more rigid
 - ▶ Differences from the crystal structure: crystallization artefacts?

Reliability of dummy atom models

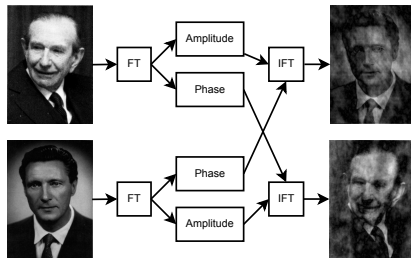
- ▶ Phase problem!
- ▶ Methods to improve reliability
 - ▶ Several candidate shapes from multiple runs of DAMMIF
 - ▶ Screening the candidates with DAMSEL
 - ▶ Average the remaining shapes with DAMAVER
 - ▶ Refine the average shape with DAMMIN



Idea from Saldin et al. J. Phys.: Condens. Matter 13 (2001) 10699-10707

Reliability of dummy atom models

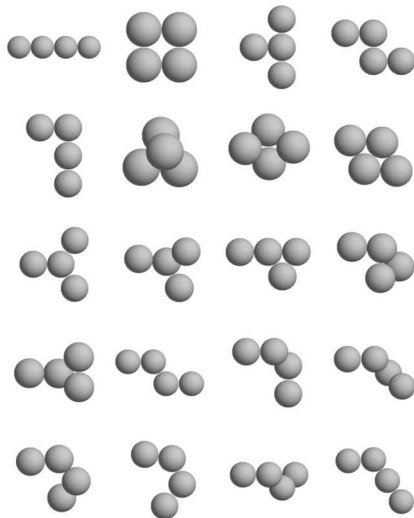
- ▶ Phase problem!
- ▶ Methods to improve reliability
 - ▶ Several candidate shapes from multiple runs of DAMMIF
 - ▶ Screening the candidates with DAMSEL
 - ▶ Average the remaining shapes with DAMAVER
 - ▶ Refine the average shape with DAMMIN
- ▶ Quantification of the ambiguity (AMBIMETER)



Idea from Saldin et al. J. Phys.: Condens. Matter 13 (2001) 10699-10707

Reliability of dummy atom models

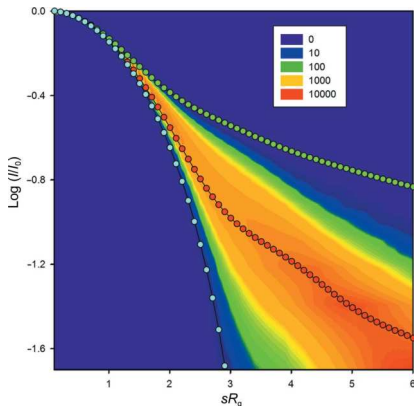
- ▶ Phase problem!
- ▶ Methods to improve reliability
 - ▶ Several candidate shapes from multiple runs of DAMMIF
 - ▶ Screening the candidates with DAMSEL
 - ▶ Average the remaining shapes with DAMAVER
 - ▶ Refine the average shape with DAMMIN
- ▶ Quantification of the ambiguity (AMBIMETER)
 - ▶ A library has been made from *all possible shapes*
 - ▶ Dimensionless scattering curves for the library elements: $I(q)/I_0$ vs. qR_g



Petoukhov & Svergun, Acta Crystallographica D 2015, 71(5), 1051-1058

Reliability of dummy atom models

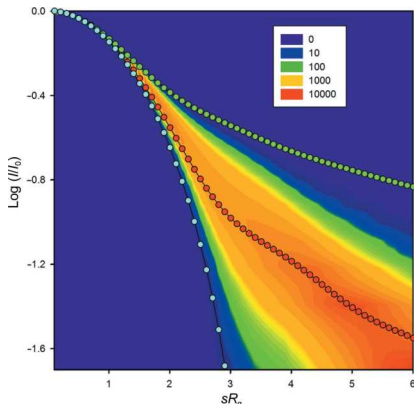
- ▶ Phase problem!
- ▶ Methods to improve reliability
 - ▶ Several candidate shapes from multiple runs of DAMMIF
 - ▶ Screening the candidates with DAMSEL
 - ▶ Average the remaining shapes with DAMAVER
 - ▶ Refine the average shape with DAMMIN
- ▶ Quantification of the ambiguity (AMBIMETER)
 - ▶ A library has been made from *all possible shapes*
 - ▶ Dimensionless scattering curves for the library elements: $I(q)/I_0$ vs. qR_g
 - ▶ Find number of those library elements where the curve is compatible with the measured one



Petoukhov & Svergun, Acta Crystallographica D 2015, 71(5), 1051-1058

Reliability of dummy atom models

- ▶ Phase problem!
- ▶ Methods to improve reliability
 - ▶ Several candidate shapes from multiple runs of DAMMIF
 - ▶ Screening the candidates with DAMSEL
 - ▶ Average the remaining shapes with DAMAVER
 - ▶ Refine the average shape with DAMMIN
- ▶ Quantification of the ambiguity (AMBIMETER)
 - ▶ A library has been made from *all possible shapes*
 - ▶ Dimensionless scattering curves for the library elements: $I(q)/I_0$ vs. qR_g
 - ▶ Find number of those library elements where the curve is compatible with the measured one
 - ▶ Lysozyme: 1; apo calmodulin: 422; Ca^{2+} -bound calmodulin: 417



Petoukhov & Svergun, Acta Crystallographica D 2015, 71(5), 1051-1058

Outline

Recapitulation

Self-assembled systems

- Ordered phases

 - Multilamellar and hexagonal phases

- Unilamellar vesicles

- Micellar systems

Continuous – hierarchical – systems

- Anisotropy and porosity of activated carbons

- In situ experiments on a gold-cysteine self-assembling nanocomplex

Particulate systems

- Size distribution of SiO₂ nanoparticles

- Proteins – biological macromolecules

Summary

Literature and software

Software

- ▶ SASFit: model fitting
- ▶ ATSAS: BioSAXS data handling, R_G , PDDF calculation, dummy atom fitting etc.
- ▶ SANSView: plotting, model fitting

Literature

- ▶ Boualem Hammouda: *Probing Nanoscale Structures: The SANS Toolbox* (http://www.ncnr.nist.gov/staff/hammouda/the_SANS_toolbox.pdf)
- ▶ J. Kohlbrecher, I. Breßler: *SASFit manual* (<http://kur.web.psi.ch/sans1/SANSSoft/sasfit.html>)
- ▶ L. A. Feigin és D. I. Svergun: *Structure Analysis by Small-Angle X-Ray and Neutron Scattering* (http://www.embl-hamburg.de/biosaxs/reprints/feigin_svergun_1987.pdf)

Summary

Interpretation of SAXS results

- ▶ Multilamellar vesicles and ordered lipid systems: determination of the periodic repeat distance
- ▶ Sterically stabilized vesicles: the radial electron density distribution of the phospholipid bilayer
- ▶ Micelles and bicelles: shape, core-shell model parameters
- ▶ Activated carbons: anisotropy, fractal properties
- ▶ Gold-cysteine nanocomplex: the time evolution of the photoluminescent nanostructure
- ▶ SiO₂ nanoparticles (repeated): size, size distribution
- ▶ BioSAXS: determination of the size, shape and flexibility of proteins *in solution*

Acknowledgements

The experimental results presented in this lecture could not have been made without the contributions (sample preparations, ideas etc.) of the following people:

- ▶ Attila Bóta
- ▶ Judith Mihály
- ▶ Zoltán Varga
- ▶ Andrea Jónás
- ▶ Andrea Bodor
- ▶ Erika Dudás
- ▶ Tünde Juhász
- ▶ Balázs Söptei

Thank you for your attention!

